ISW effect in Unified Dark Matter Scalar Field Cosmologies: an analytical approach

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Abstract. We perform an analytical study of the Integrated Sachs-Wolfe (ISW) effect within the framework of Unified Dark Matter models based on a scalar field which aim at a unified description of dark energy and dark matter. Computing the temperature power spectrum of the Cosmic Microwave Background anisotropies we are able to isolate those contributions that can potentially lead to strong deviations from the usual ISW effect occurring in a ΛCDM universe. This helps to highlight the crucial role played by the sound speed in the unified dark matter models. Our treatment is completely general in that all the results depend only on the speed of sound of the dark component and thus it can be applied to a variety of unified models, including those which are not described by a scalar field but relies on a single dark fluid.

1. Introduction

Observations of large scale structure, search for Ia supernovae (SNIa), measurements of the Cosmic Microwave Background (CMB) anisotropies all suggest that two dark components govern the dynamics of the universe. In particular they are the dark matter (DM), responsible for structure formation, and an additional dark energy component that drives the cosmic acceleration observed at present [2, 3]. Two main routes have been followed to provide a plausible realization of the dark energy, a non-zero cosmological constant Λ (see, e.g. Ref. [1]), or a dynamical dark energy (DE) component in the form of a scalar field, like for example *Quintessence* [4, 5, 6, 7, 8, 9, 10, 11] and *k-essence* models [12, 13, 14]. The latter are characterized by a Lagrangian with non-canonical kinetic term and inspired by earlier studies of k-inflation [15] [16] (a complete list of dark energy models can be found in the recent review [17]). At different levels all of these scenarios suffer of non trivial fine tuning problems and of the so called cosmic coincidence problem (why Ω_{DM} and Ω_Λ are both of order unity today).

More recently the alternative hypothesis of unified models of dark energy and dark matter has been considered. In this case a single fluid behaves both as dark matter and dark energy. This has been variously referred to as "Unified Dark Matter" (UDM), or "Quartessence". Among several models of k-essence considered in the literature there exist several UDM models. First of all *purely kinetic models*, i.e. the *generalized Chaplygin gas* (GCG) [18, 19, 20] model, the Scherrer and generalized Scherrer solutions [21] [22], a single dark perfect fluid with a simple 2-parameter barotropic equation of state [23], or the homogeneous scalar field deduced from the galactic halo space-times [24]. Moreover there are models in which imposing the Lagrangian of the scalar field to be a constant allows directly to describe a unified dark matter/ dark energy fluid [22] [25] (see also, with different approach, [26]). Alternative approaches to the unification of DM and DE have been proposed in Ref. [27], in the frame of supersymmetry, and in Ref. [28], in connection with the solution of the strong CP problem.

The recent attitude in analysing the observational consequences of the DE models has been that of considering not only the background equation of state and its evolution with time, but also to focus on the sound speed which regulates the growth of the dark energy fluid perturbations on different cosmological scales. In this case the sound speed has been often treated as a completely independent parameter in order to explore the consequences on the CMB anisotropies and its effects on the low ℓ multipoles [29, 30, 31]. The efficiency of this method relies on the observation that, for a single scalar field with canonical kinetic term, the speed of sound is equal to the speed of light, and thus it can cluster only on scales of the horizon size, while for other models it can be lower than unity, implying the possibility of clustering on smaller scales [32]. Another important issue is whether the dark matter clustering is influenced by the dark energy and, for the unified models, it becomes especially relevant in view of this approach. In the GCG model (both as dark energy and unified dark matter) strong constraints come from the CMB anisotropies [33, 34, 35] and the analysis of the mass power spectrum [36]. In the Scherrer solution the parameters of the model have to be fine-tuned in order for the model not to exhibit finite pressure effects in the non-linear stages of structure formation [37].

In this paper we consider cosmological models where dark matter and dark energy are manifestations of a single scalar field, and we focus on the contribution to the large-scale CMB anisotropies which is due to the evolution in time of the gravitational potential from the epoch of last scattering up now, the so called late Integrated Sachs-Wolfe (ISW) effect [38]. Through an analytical approach we point out the crucial role of the speed of sound in the unified dark matter models in determining strong deviations from the usual standard ISW occurring in the ΛCDM models. Our treatment is completely general in that all the results depend only on the speed of sound of the dark component and thus it can be applied to a variety of models, including those which are not described by a scalar field but relies on a single perfect dark fluid. In the case of ΛCDM models the ISW is dictated by the background evolution, which causes the late time decay of the gravitational potential when the cosmological constant starts to dominate [39]. In the case of the unified models there are two simple but important aspects: first, the fluid which triggers the accelerated expansion at late times is also the one which has to cluster in order to produce the structures we see today. Second, from the last scattering to the present epoch, the energy density of the universe is dominated by a single dark fluid, and therefore the gravitational potential evolution is determined by the background and perturbation evolution of just such a fluid. As a result the general trend is that the possible appearance of a sound speed significantly different from zero at late times corresponds to the appearance of a Jeans length (or a sound horizon) under which the dark fluid does not cluster any more, causing a strong evolution in time of the gravitational potential (which starts to oscillate and decay) and thus a strong ISW effect. Our results show explicitly that the CMB temperature power spectrum C_ℓ for the ISW effect contains some terms depending on the speed of sound which give a high contribution along a wide range of multipoles ℓ. As the most straightforward way to avoid these critical terms one can require the sound speed to be always very close to zero (thou see Sec. 3.2.3 for a more detailed discussion on this point). Moreover we find that such strong imprints from the ISW effect comes primarily from the evolution of the dark component perturbations, rather than from the background expansion history. The paper is organized as follows. In Sec. 2 we obtain the evolution equation for the gravitational potential. In Sec. 3 we start the analytical analysis of the ISW effect, dividing the resulting expression for the angular CMB power spectrum according to three relevant regions: those perturbation modes that enter the horizon after the acceleration of the universe becomes relevant, and perturbation modes that are inside or outside the sound horizon of the dark fluid. In Sec. 3.2.2 we point out those contributions to the ISW effect that are triggered by the sound speed and that are responsible for a strong ISW imprint. Sec. 4 contains our conclusions and a discussion of our results applied to various unified dark matter models.

2. Linear perturbations in scalar field unified dark matter models

We consider the action that describes most of the dark matter unified models within the framework of k-essence

$$S = S_G + S_\varphi = \int d^4x\sqrt{-g}\left[\frac{R}{2} + \mathcal{L}(\varphi, X)\right] \tag{1}$$

where

$$X = -\frac{1}{2}\nabla_\mu\varphi\nabla^\mu\varphi \ . \tag{2}$$

We use units such that $8\pi G = 1$ and signature $(-,+,+,+)$.

The energy-momentum tensor of the scalar field φ is

$$T^\varphi_{\mu\nu} = -\frac{2}{\sqrt{-g}}\frac{\delta S_\varphi}{\delta g^{\mu\nu}} = \frac{\partial\mathcal{L}(\varphi, X)}{\partial X}\nabla_\mu\varphi\nabla_\nu\varphi + \mathcal{L}(\varphi, X)g_{\mu\nu}. \tag{3}$$

If X is time-like S_φ describes a perfect fluid with $T^\varphi_{\mu\nu} = (\rho+p)u_\mu u_\nu + p\, g_{\mu\nu}$, with pressure

$$\mathcal{L} = p(\varphi, X) \ , \tag{4}$$

and energy density

$$\rho = \rho(\varphi, X) = 2X\frac{\partial p(\varphi, X)}{\partial X} - p(\varphi, X) \tag{5}$$

where

$$u_\mu = \frac{\nabla_\mu\varphi}{\sqrt{2X}}. \tag{6}$$

the four-velocity.

Now we assume a flat, homogeneous Friedmann-Robertson-Walker background metric i.e.

$$ds^2 = -dt^2 + a(t)^2\delta_{ij}dx^i dx^j = a(\eta)^2(-d\eta^2 + \delta_{ij}dx^i dx^j)\,, \tag{7}$$

where $a(t)$ is the scale factor,δ_{ij} denotes the unit tensor and $d\eta = dt/a$ is the conformal time. In such a case, the background evolution of the universe is characterized completely by the following equations

$$\mathcal{H}^2 = a^2H^2 = \frac{1}{3}a^2\rho\,, \tag{8}$$

$$\mathcal{H}' - \mathcal{H}^2 = a^2\dot{H} = -\frac{1}{2}a^2(p+\rho)\,, \tag{9}$$

where $\mathcal{H} = a'/a$, the dot denotes differentiation w.r.t. the cosmic time t and a prime w.r.t. the conformal time η. On the background $X = \frac{1}{2}\dot{\varphi}^2 = \varphi'^2/(2a^2)$ and the equation of motion for the homogeneous mode $\varphi(t)$ becomes

$$\left(\frac{\partial p}{\partial X} + 2X\frac{\partial^2 p}{\partial X^2}\right)\ddot{\varphi} + \frac{\partial p}{\partial X}(3H\dot{\varphi}) + \frac{\partial^2 p}{\partial\varphi\partial X}\dot{\varphi}^2 - \frac{\partial p}{\partial\varphi} = 0\ . \tag{10}$$

One of the relevant quantities for the dark energy issue is the equation of state $w \equiv p/\rho$ which in our case reads

$$w = \frac{p}{2X\frac{\partial p}{\partial X} - p}\ . \tag{11}$$

On the other hand we will focus on the other relevant physical quantity, the speed of sound, which enters in governing the evolution of the scalar field perturbations. Considering small inhomogeneities of the scalar field

$$\varphi(t,x) = \varphi_0(t) + \delta\varphi(t,\boldsymbol{x}) , \tag{12}$$

we can write the metric in the longitudinal gauge as

$$ds^2 = -(1+2\Phi)dt^2 + (1-2\Phi)a(t)^2\delta_{ij}dx^i dx^j \tag{13}$$

since $\delta T_i^j = 0$ for $i \neq j$ [40].

From the linearized $(0-0)$ and $(0-i)$ Einstein equation one obtains (see Ref. [16] and Ref. [41])

$$\nabla^2\Phi = \frac{1}{2}\frac{a^2(p+\rho)}{c_s^2\mathcal{H}}\left(\mathcal{H}\frac{\delta\varphi}{\varphi_0'} + \Phi\right)' , \tag{14}$$

$$\left(a^2\frac{\Phi}{\mathcal{H}}\right)' = \frac{1}{2}\frac{a^2(p+\rho)}{\mathcal{H}^2}\left(\mathcal{H}\frac{\delta\varphi}{\varphi_0'} + \Phi\right) , \tag{15}$$

where one defines a "speed of sound" c_s^2 relative to the pressure and energy density fluctuation of the kinetic term [16] as

$$c_s^2 \equiv \frac{(\partial p/\partial X)}{(\partial\rho/\partial X)} = \frac{\frac{\partial p}{\partial X}}{\frac{\partial p}{\partial X} + 2X\frac{\partial^2 p}{\partial X^2}} . \tag{16}$$

Eqs. (14) and (15) are sufficient to determine the gravitational potential Φ and the perturbation of the scalar field. Defining two new variables

$$u \equiv 2\frac{\Phi}{(p+\rho)^{1/2}} , \qquad v \equiv z\left(\mathcal{H}\frac{\delta\varphi}{\varphi_0'} + \Phi\right) , \tag{17}$$

where $z = a^2(p+\rho)^{1/2}/(c_s\mathcal{H})$, we can recast (14) and (15) in terms of u and v [41]

$$c_s\triangle u = z\left(\frac{v}{z}\right)' , \qquad c_s v = \theta\left(\frac{u}{\theta}\right)' \tag{18}$$

where $\theta = 1/(c_s z) = (1+p/\rho)^{-1/2}/(\sqrt{3}a)$. Starting from (18) we arrive at the following second order differential equations for u [41]

$$u'' - c_s^2\nabla^2 u - \frac{\theta''}{\theta}u = 0 . \tag{19}$$

Notice that this equation can also be used to describe any perfect fluid with equation of state $p = p(\rho)$, up to a redefinition of c_s. In this case $c_s^2 = p'/\rho'$ corresponds to the usual adiabatic sound speed. In this way, with the same equation (19), we can also describe the ΛCDM model. Also pure kinetic Lagrangian (4) $\mathcal{L}(X)$ models (see for example Ref. [22]), can be described as a perfect fluid with the pressure p uniquely determined by the energy density, since they both depend on a single degree of freedom, the kinetc term X.

Unfortunately we do not know the exact solution for a generic Lagrangian. However we can consider the asymptotic solutions i.e. long-wavelength and short-wavelength perturbations, depending whether $c_s^2k^2 \ll |\theta''/\theta|$ or $c_s^2k^2 \gg |\theta''/\theta|$, respectively. This means to consider perturbations on scale much larger or much smaller than the effective Jeans length for the gravitational potential $\lambda_J^2 = c_s^2\,|\theta/\theta''|$.

For a plane wave perturbation $u \propto u_k(\eta)\exp(i\boldsymbol{k}\boldsymbol{x})$ in the short-wavelength limit ($c_s^2 k^2 \gg |\theta''/\theta|$) we obtain

$$u_k \simeq \frac{C_k(\bar\eta)}{c_s^{1/2}(\eta)}\cos\left(k\int_{\bar\eta}^{\eta} c_s d\tilde\eta\right)\,, \tag{20}$$

where C_k is a constant of integration. Instead, neglecting the decaying mode, the long-wavelength solution ($c_s^2 k^2 \ll |\theta''/\theta|$) is

$$u_k = A_k(\bar\eta)\theta\int_{\bar\eta}^{\eta}\frac{d\tilde\eta}{\theta^2}\,, \tag{21}$$

where A_k is a constant of integration .
Once u is computed we can obtain the value of the gravitational potential Φ through Eq. (17) and the perturbation of the scalar field from Eq. (15)

$$\delta\varphi = 2\sqrt{2X}\frac{(\Phi' + \mathcal{H}\Phi)}{a(p+\rho)}\,. \tag{22}$$

3. Analytical approach to the ISW effect

Let us now focus on the ISW effect. The ISW contribution to the CMB power spectrum is given by

$$\frac{2l+1}{4\pi}C_l^{\rm ISW} = \frac{1}{2\pi^2}\int_0^\infty \frac{dk}{k}k^3\frac{\left|\Theta_l^{\rm ISW}(\eta_0,k)\right|^2}{2l+1}\,, \tag{23}$$

where $\Theta_l^{\rm ISW}$ is the fractional temperature perturbation due to ISW effect

$$\frac{\Theta_l^{\rm ISW}(\eta_0,k)}{2l+1} = 2\int_{\eta_*}^{\eta_0}\Phi'(\tilde\eta,k)j_l[k(\eta_0-\tilde\eta)]d\tilde\eta\,, \tag{24}$$

with η_0 and η_* the present and the last scattering conformal times respectively and j_l are the spherical Bessel functions.
We now evaluate analytically the power spectrum (23). As a first step, following the same procedure of Ref. [39], we notice that, when the acceleration of the universe begins to be important, the expansion time scale $\eta_{1/2} = \eta(w = -1/2)$ sets a critical wavelength corresponding to $k\eta_{1/2} = 1$. It is easy to see that if we consider the ΛCDM model then $\eta_{1/2} = \eta_\Lambda$ i.e. when $a_\Lambda/a_0 = (\Omega_0/\Omega_\Lambda)^{1/3}$ [39]. Thus at this critical point we can break the integral (23) in two parts [39]

$$\frac{2l+1}{4\pi}C_l^{\rm ISW} = \frac{1}{2\pi^2}\left[I_{\Theta_l}(k\eta_{1/2}<1) + I_{\Theta_l}(k\eta_{1/2}>1)\right]\,, \tag{25}$$

where

$$I_{\Theta_l}(k\eta_{1/2}<1) \equiv \int_0^{1/\eta_{1/2}}\frac{dk}{k}k^3\frac{\left|\Theta_l^{\rm ISW}(\eta_0,k)\right|^2}{2l+1}\,, \tag{26}$$

and

$$I_{\Theta_l}(k\eta_{1/2}>1) \equiv \int_{1/\eta_{1/2}}^{\infty}\frac{dk}{k}k^3\frac{\left|\Theta_l^{\rm ISW}(\eta_0,k)\right|^2}{2l+1}\,. \tag{27}$$

As explained in Ref. [39] the ISW integrals (24) takes on different forms in these two regimes

$$\frac{\Theta_{l\,\mathrm{ISW}}(\eta_0,k)}{2l+1} = \begin{cases} 2\Delta\Phi_k\; j_l[k(\eta_0-\eta_{1/2})] & k\eta_{1/2}\ll 1 \\ 2\Phi'_k(\eta_k)I_l/k & k\eta_{1/2}\gg 1 \end{cases} \tag{28}$$

where $\Delta\Phi_k$ is the change in the potential from the matter-dominated (for example at recombination) to the present epoch η_0 and $\eta_k \simeq \eta_0 - (l+1/2)/k$ is the conformal time when a given k-mode contributes maximally to the angle that this scale subtends on the sky, obtained at the peak of the Bessel fucntion j_ℓ. The first limit in Eq. (28) is obtained by approximating the Bessel function as a constant evaluated at the critical epoch $\eta_{1/2}$. Since it comes from perturbations of wavelenghts longer than the distance a photon can travel during the the time $\eta_{1/2}$, a kick ($2\Delta\Phi_k$) to the photons is the main result, and it will corresponds to very low multipoles, since $\eta_{1/2}$ is very close to the present epoch η_0. It thus appears similar to a Sachs-wolfe effect (or also to the early ISW contribution). The second limit in Eq. (28) is achieved by considering the strong oscillations of the Bessel functions in this regime, and thus evaluating the time derivative of the potentials out of the integral at the peak of the Bessel function, leaving the integral [39]

$$I_l \equiv \int_0^\infty j_l(y)dy = \frac{\sqrt{\pi}}{2}\frac{\Gamma[(l+1)/2]}{\Gamma[(l+2)/2]}\;. \tag{29}$$

With this procedure, replacing (28a) in (26) and (28b) in (27) we can obtain the ISW contribution to the CMB anisotropies power spectrum (23).

Now we have to calculate, through Eqs. (20)-(21) and (17), the value of $\Phi(k,\eta)$ for $k\eta_{1/2}\ll 1$ and $k\eta_{1/2}\gg 1$. As we will see tha main differences (and the main difficulties) of the unified dark matter models with respect to the ΛCDM case will appear from the second regime of Eq. (28).

3.1. Derivation of I_{Θ_l} for modes $k\eta_{1/2} < 1$

In the UDM models when $k\eta_{1/2}\ll 1$ then $c_s^2k^2 \ll |\theta''/\theta|$ is always satisfied. This is due to the fact that before the dark fluid start to be relevant as a cosmological constant, for $\eta < \eta_{1/2}$, its sound speed generically is very close to zero in order to guarantee enough structure formation, and moreover the limit $k\eta_{1/2}\ll 1$ involves very large scales (since $\eta_{1/2}$ is very close to the present epoch). For the standard ΛCDM model the condition is clearly satisifed. In this situation we can use the relation (21) and Φ_k becomes

$$\Phi_k = A_k\left(1-\frac{\mathcal{H}(\eta)}{a^2(\eta)}\int_{\eta_i}^{\eta} a^2(\tilde\eta)d\tilde\eta\right)\;. \tag{30}$$

We immediately see that $A_k = \Phi_k(0)$, the large scale gravitational potential during the radiation dominated epoch. The integral in Eq. (30) may be written as follows

$$\int_{\eta_i}^{\eta} a^2(\tilde\eta)d\tilde\eta = I_R + \int_{\eta_R}^{\eta} a^2(\tilde\eta)d\tilde\eta\,, \tag{31}$$

where $I_R = \int_{\eta_i}^{\eta_R} a^2(\tilde{\eta}) d\tilde{\eta}$ and η_R is the conformal time at recombination. When $\eta_i < \eta < \eta_R$ the UDM Models behave as dark matter ‡. In this temporal range the universe is dominated by a mixture of "matter" and radiation and I_R becomes

$$I_R = \eta_* a_{eq} \left(\frac{\xi_R^5}{5} + \xi_R^4 + \frac{4\xi_R^3}{3} \right) \tag{32}$$

where a_{eq} is the value of the scalar factor at matter-radiation equality, $\xi = \eta/\eta_*$ and $\eta_* = (\rho_{eq} a_{eq}^2/24)^{-1/2} = \eta_{eq}/(\sqrt{2}-1)$. With these definitions it is easy to see that $a_R = a_{eq}(\xi_R^2 + 2\xi_R)$. Notice that Eq. (30) is obtained in the case of adiabatic perturbations. Since we are dealing with unified dark matter models based on a scalar field, there will always be an intrinsic non-adiabatic pressure (or entropic) pertubation. However for the very long wavelenghts, $k\eta_{1/2} \ll 1$ under consideration here such an intrinsic perturbation turns out to be negligible [16]. For adiabatic perturbations $\Phi_k(\eta_R) \cong (9/10)\Phi_k(0)$ [40] and accounting for the primordial power spectrum, $k^3|\Phi_k(0)|^2 = Bk^{n-1}$, where n is the scalar spectral index, we get from Eq. (28a)

$$I_{\Theta_l}(k\eta_{1/2} < 1) \approx 4(2l+1)B \int_0^{1/\eta_{1/2}} \frac{dk}{k} k^{n-1} j_l^2[k(\eta_0 - \eta_{1/2})] \times \left| \frac{1}{10} - \frac{\mathcal{H}(\eta_0)}{a^2(\eta_0)} \left[\int_{\eta_R}^{\eta_0} a^2(\tilde{\eta}) d\tilde{\eta} \right] \right|^2 , \tag{33}$$

where we have neglected I_R since it gives a negligible contribution.

A first comment is in order here. There is a vast class of unified dark matter models that are able to reproduce exactly the same background expansion history of the universe as the ΛCDM model (at least from the recombination epoch onwards). For example this is the case of the the Scherrer and generalized Scherrer unified models [21] [22], the generalized Chaplygin gas [18, 19, 20] for the parameter α which tends to zero, the models proposed in Ref. [22] and [25] where one impose the langrangian (i.e. the pressure) to be a constant, and also the model of a single dark perfect fluid proposed in Ref. [23]. For such cases it is clear that the low ℓ contribution (33) to the ISW effect will be the same that is predicted by the ΛCDM model. This is easily explained considering that for such long wevelenght perturbations the sound speed in fact plays no role at all.

3.2. Derivation of I_{Θ_l} for modes $k\eta_{1/2} > 1$

As we have already mentioned in the previous section, in general a viable UDM must have a sound speed very close to zero for $\eta < \eta_{1/2}$ in order to behave as dark matter also at the perturbed level to form the structures we see today, and thus the gravitational potential will start to change in time for $\eta > \eta_{1/2}$. Therefore for the modes $k\eta_{1/2} > 1$, in order to evaluate Eq. (28b) into Eq. (27) we can impose that $\eta_k > \eta_{1/2}$ which, from the definition of $\eta_k \simeq \eta_0 - (l+1/2)/k$, moves the lower limit of Eq. (27) to $(l+1/2)/(\eta_0 - \eta_{1/2})$.

‡ In fact the Scherrer [21] and generalized Scherrer solutions [22] in the very early universe, much before the equality epoch, have $c_s \neq 0$ and $w > 0$. However at these times the dark fluid contribution is sub-dominant with respect to the radiation energy density and thus ther is no substantial effect on the following equations.

Moreover we have that $\eta_{1/2} \sim \eta_0$. We can use this property to estimate any observable at the value of η_k. Defining

$$\chi = \frac{\eta}{\eta_{1/2}}\,, \qquad \text{and} \qquad \kappa = k\eta_{1/2}\,, \tag{34}$$

we have

$$a_k = a(\eta_k) = a(\chi_k) = a_0 + \left.\frac{da}{d\chi}\right|_{\chi_0} \delta\chi_k = 1 - \eta_{1/2}\mathcal{H}_0 \frac{l+1/2}{\kappa}\,, \tag{35}$$

taking $a_0 = 1$, and

$$\frac{d\Phi_k}{d\chi}(\chi_k) = \eta_{1/2}\Phi'(\eta_k) = \left.\frac{d\Phi_k}{d\chi}\right|_{\chi_0} - \left.\frac{d^2\Phi_k}{d\chi^2}\right|_{\chi_0} \left(\frac{l+1/2}{\kappa}\right)\,, \tag{36}$$

where $\delta\chi_k = \chi_k - \chi_0 = (\eta_k - \eta_0)/\eta_{1/2} = -(l+1/2)/\kappa$. Notice that the expansion (36) is fully justified, since as already mentioned above, the mimimum value of κ in Eq. (27) moves to $(l+1/2)/(\eta_0/\eta_{1/2} - 1)$, making $\delta\chi_k$ much less than 1. Therefore we can write

$$\begin{aligned}\frac{|\Theta_{l\,\mathrm{ISW}}(\eta_0,k)|^2}{(2l+1)^2} &= 4\left|\frac{\Phi'_k(\eta_k)I_l}{k}\right|^2 = \frac{4I_l^2}{\kappa^2}\left|\frac{d\Phi_k}{d\chi}(\chi_k)\right|^2 = \\ &= \frac{4I_l^2}{\kappa^2}\left[\left|\frac{d\Phi_k}{d\chi}(\chi_0)\right|^2 - 2\frac{d\Phi_k}{d\chi}(\chi_0)\frac{d^2\Phi_k}{d\chi^2}(\chi_0)\left(\frac{l+1/2}{\kappa}\right) + \right. \\ &\left. + \left|\frac{d^2\Phi_k}{d\chi^2}(\chi_0)\right|^2\left(\frac{l+1/2}{\kappa}\right)^2\right]\,.\end{aligned} \tag{37}$$

In this case, during $\eta_{1/2} < \eta < \eta_0$, there will be perturbation modes whose wavelength stays bigger than the Jeans length or smaller than it, i.e. we have to consider both the possibilities $c_s^2k^2 \ll |\theta''/\theta|$ and $c_s^2k^2 \gg |\theta''/\theta|$. In general the sound speed can vary with time, and in particular it might become significantly different from zero at late times. However, just as a first approximation, we exclude the intermediate situation because usually $\eta_{1/2}$ is very close to η_0 (this situation will be briefly analyzed later).

3.2.1. Perturbation modes on scales bigger than the Jeans length.

When $c_s^2k^2 \ll |\theta''/\theta|$ the value of $\Phi'(\eta_k)$ can be written from Eq.(30) as

$$\Phi'(\eta_k) = \Phi_k(0)\tilde{\Phi}'_k(\eta_k) = \Phi_k(0)a(\eta_k)\left[\frac{d^2}{dt^2}\left(\frac{1}{a}\int_{t_i}^t a(\tilde{t})d\tilde{t}\right)\right]_{t=t(\eta_k)}\,. \tag{38}$$

Now, using this expression in Eq. (37), with the primordial power spectrum $k^3|\Phi_k(0)|^2 = Bk^{n-1}$, the value of (27) may be written as

$$\begin{aligned}\frac{I_{\Theta_l}(k\eta_{1/2} > 1)}{2l+1} &= 4I_l^2 B\eta_{1/2}^{n-1}\left[\int_{\frac{l+1/2}{\chi_0-1}}^{\infty}\frac{d\kappa}{\kappa^3}\kappa^{n-1}\left|\frac{d\tilde{\Phi}_k}{d\chi}(\chi_k)\right|^2\right] = \\ &= 4I_l^2 B\eta_{1/2}^{n-1}\left[\frac{1}{3-n}\left(\frac{\chi_0-1}{l+1/2}\right)^{3-n}\left|\frac{d\tilde{\Phi}_k}{d\chi}(\chi_0)\right|^2 + \right.\end{aligned}$$

$$- \frac{2(l+1/2)}{4-n}\left(\frac{\chi_0-1}{l+1/2}\right)^{4-n}\frac{d\tilde{\Phi}_k}{d\chi}(\chi_0)\frac{d^2\tilde{\Phi}_k}{d\chi^2}(\chi_0)+$$
$$+ \frac{(l+1/2)^2}{5-n}\left(\frac{\chi_0-1}{l+1/2}\right)^{5-n}\left|\frac{d^2\tilde{\Phi}_k}{d\chi^2}(\chi_0)\right|^2\Bigg] \quad (39)$$

with $(d\tilde{\Phi}_k/d\chi)_{\chi_0} = \eta_{1/2}\tilde{\Phi}'_k(\eta_0)$ and with $(d^2\tilde{\Phi}_k/d\chi^2)_{\chi_0} = \eta_{1/2}^2\tilde{\Phi}''_k(\eta_0)$.
A second relevant comment follows from the fact that $I_l^2 \sim 1/l$ for $l \gg 1$. We thus see that for $n=1$ and for $l \gg 1$ the contribution to the angular power spectrum from the modes under consideration is $l(l+1)C_l^{ISW}/(4\pi) = l(l+1)I_{\Theta_l}(k\eta_{1/2} > 1)/(2\pi^2(2l+1)) \sim 1/l$. In other words we find a similar slope as in [39, 42] found in the ΛCDM model. Recalling the results of the previous section, this means that in the unified dark matter models the contribution to the ISW effect from those perturbations that are outside the Jeans length is very similar to the one produced in a ΛCDM model. The main difference on these scales will be present if the background evolution is different from the one in the ΛCDM model, but for the models where the background evolution is the same, as those proposed in Refs. [21, 22, 25, 23] no difference at all can be observable.

3.2.2. Perturbation modes on scales smaller than the Jeans length.

When $c_s^2k^2 \gg |\theta''/\theta|$ we must use the solution (20) and through the relation (17a) the gravitational potential is given by

$$\Phi_k(\eta) = \frac{1}{2}\left[(p+\rho)/c_s\right]^{1/2}(\eta)\, C_k(\eta_{1/2})\cos\left(k\int_{\eta_{1/2}}^{\eta} c_s(\tilde{\eta})d\tilde{\eta}\right). \quad (40)$$

In Eq. (40) $C_k(\eta_{1/2}) = \Phi_k(0)C_{1/2}$ is a constant of integration where

$$C_{1/2} = 2\frac{\left[1 - \frac{\mathcal{H}(\eta_{1/2})}{a^2(\eta_{1/2})}\left(I_R + \int_{\eta_R}^{\eta_{1/2}} a^2(\tilde{\eta})d\tilde{\eta}\right)\right]}{\left[(p+\rho)/c_s\right]^{1/2}(\eta_{1/2})}, \quad (41)$$

and it is obtained under the approximation that for $\eta < \eta_{1/2}$ one can use the longwavelenght solution (30), since for these epochs the sound speed must be very close to zero. Notice that Eq. (40) shows clearly that the gravitational potential is oscillating and decaying in time.

Defining for simplicity $\overline{C}^2 = C_{1/2}^2[(p+\rho)/c_s](\eta_0)/4$, we take the time derivative of the gravitational potential appearing in Eq. (28b) by employing the expansion of Eq.(37). We thus find that Eq. (27) yields

$$\frac{I_{\Theta_l}(k\eta_{1/2} > 1)}{2l+1} = 4\overline{C}^2 B I_l^2 \eta_{1/2}^{n-1}\Bigg\{\mathcal{C}_{\{k5,l2,c^2\}}(l+1/2)^2\left[\int_{\frac{l+1/2}{\chi_0-1}}^{\infty}\frac{d\kappa}{\kappa^5}\kappa^{n-1}\cos^2(D_0\kappa)\right]+$$
$$+ \mathcal{C}_{\{k4,l1,c^2\}}(l+1/2)\left[\int_{\frac{l+1/2}{\chi_0-1}}^{\infty}\frac{d\kappa}{\kappa^4}\kappa^{n-1}\cos^2(D_0\kappa)\right]+$$
$$+ \mathcal{C}_{\{k4,l2,sc\}}(l+1/2)^2\left[\int_{\frac{l+1/2}{\chi_0-1}}^{\infty}\frac{d\kappa}{\kappa^4}\kappa^{n-1}\cos(D_0\kappa)\sin(D_0\kappa)\right]+$$
$$+ \left[\mathcal{C}_{\{k3,l0,c^2\}} + \mathcal{C}_{\{k3,l2,c^2\}}(l+1/2)^2\right]\left[\int_{\frac{l+1/2}{\chi_0-1}}^{\infty}\frac{d\kappa}{\kappa^3}\kappa^{n-1}\cos^2(D_0\kappa)\right]+$$

$$
\begin{aligned}
&+\mathcal{C}_{\{k3,l2,s^2\}}(l+1/2)^2\left[\int_{\frac{l+1/2}{\chi_0-1}}^{\infty}\frac{d\kappa}{\kappa^3}\kappa^{n-1}\sin^2(D_0\kappa)\right]\\
&+\mathcal{C}_{\{k3,l1,sc\}}(l+1/2)\left[\int_{\frac{l+1/2}{\chi_0-1}}^{\infty}\frac{d\kappa}{\kappa^3}\kappa^{n-1}\cos(D_0\kappa)\sin(D_0\kappa)\right]+\\
&+\mathcal{C}_{\{k2,l1,c^2\}}(l+1/2)\left[\int_{\frac{l+1/2}{\chi_0-1}}^{\infty}\frac{d\kappa}{\kappa^2}\kappa^{n-1}\cos^2(D_0\kappa)\right]+\\
&+\mathcal{C}_{\{k2,l1,s^2\}}(l+1/2)\left[\int_{\frac{l+1/2}{\chi_0-1}}^{\infty}\frac{d\kappa}{\kappa^2}\kappa^{n-1}\sin^2(D_0\kappa)\right]+\\
&+\left[\mathcal{C}_{\{k2,l0,sc\}}+\mathcal{C}_{\{k2,l2,sc\}}(l+1/2)^2\right]\left[\int_{\frac{l+1/2}{\chi_0-1}}^{\infty}\frac{d\kappa}{\kappa^2}\kappa^{n-1}\cos(D_0\kappa)\sin(D_0\kappa)\right]+\\
&+\mathcal{C}_{\{k1,l0,s^2\}}\left[\int_{\frac{l+1/2}{\chi_0-1}}^{\infty}\frac{d\kappa}{\kappa}\kappa^{n-1}\sin^2(D_0\kappa)\right]+\\
&+\mathcal{C}_{\{k1,l2,c^2\}}(l+1/2)^2\left[\int_{\frac{l+1/2}{\chi_0-1}}^{\infty}\frac{d\kappa}{\kappa}\kappa^{n-1}\cos^2(D_0\kappa)\right]+\\
&+\mathcal{C}_{\{k1,l1,sc\}}(l+1/2)\left[\int_{\frac{l+1/2}{\chi_0-1}}^{\infty}\frac{d\kappa}{\kappa}\kappa^{n-1}\cos(D_0\kappa)\sin(D_0\kappa)\right]\Bigg\}
\end{aligned}
\tag{42}
$$

with $D_0 = \int_1^{\chi_0} c_s(\tilde{\chi})d\tilde{\chi}$ and where

$$
\mathcal{C}_{\{k5,l2,c^2\}} = \left\{\frac{(p+\rho)_{,\chi\chi}}{(p+\rho)} - \left(\frac{(p+\rho)_{,\chi}}{(p+\rho)} - \frac{c_{s,\chi}}{c_s}\right)\left[2\frac{c_{s,\chi}}{c_s} + \frac{1}{2}\left(\frac{(p+\rho)_{,\chi}}{(p+\rho)} - \frac{c_{s,\chi}}{c_s}\right)\right]\right\}^2\Bigg|_{\chi_0},
$$

$$
\begin{aligned}
\mathcal{C}_{\{k4,l1,c^2\}} = -2\Bigg\{&\left(\frac{(p+\rho)_{,\chi}}{(p+\rho)} - \frac{c_{s,\chi}}{c_s}\right)^2\Bigg[\frac{(p+\rho)_{,\chi\chi}}{(p+\rho)}\\
&-\left(\frac{(p+\rho)_{,\chi}}{(p+\rho)} - \frac{c_{s,\chi}}{c_s}\right)\left(2\frac{c_{s,\chi}}{c_s} + \frac{1}{2}\left(\frac{(p+\rho)_{,\chi}}{(p+\rho)} - \frac{c_{s,\chi}}{c_s}\right)\right)\Bigg]\Bigg\}\Bigg|_{\chi_0},
\end{aligned}
$$

$$
\begin{aligned}
\mathcal{C}_{\{k4,l2,sc\}} = 4\Bigg\{&c_s\frac{(p+\rho)_{,\chi}}{(p+\rho)}\Bigg[\frac{(p+\rho)_{,\chi\chi}}{(p+\rho)} - \left(\frac{(p+\rho)_{,\chi}}{(p+\rho)} - \frac{c_{s,\chi}}{c_s}\right)\\
&\left(2\frac{c_{s,\chi}}{c_s} + \frac{1}{2}\left(\frac{(p+\rho)_{,\chi}}{(p+\rho)} - \frac{c_{s,\chi}}{c_s}\right)\right)\Bigg]\Bigg\}\Bigg|_{\chi_0},
\end{aligned}
$$

$$
\mathcal{C}_{\{k3,l0,c^2\}} = \left[\frac{(p+\rho)_{,\chi}}{(p+\rho)} - \frac{c_{s,\chi}}{c_s}\right]^2\Bigg|_{\chi_0}, \qquad \mathcal{C}_{\{k3,l2,s^2\}} = 4\left[c_s\frac{(p+\rho)_{,\chi}}{(p+\rho)}\right]^2\Bigg|_{\chi_0},
$$

$$
\mathcal{C}_{\{k3,l2,c^2\}} = 4\left\{c_s^2\left[\frac{(p+\rho)_{,\chi\chi}}{(p+\rho)} - \left(\frac{(p+\rho)_{,\chi}}{(p+\rho)} - \frac{c_{s,\chi}}{c_s}\right)\left(2\frac{c_{s,\chi}}{c_s} + \frac{1}{2}\left(\frac{(p+\rho)_{,\chi}}{(p+\rho)} - \frac{c_{s,\chi}}{c_s}\right)\right)\right]\right\}\Bigg|_{\chi_0},
$$

$$
\begin{aligned}
\mathcal{C}_{\{k3,l1,sc\}} = 4\Bigg\{&c_s\left[\frac{(p+\rho)_{,\chi\chi}}{(p+\rho)} - \left(\frac{(p+\rho)_{,\chi}}{(p+\rho)} - \frac{c_{s,\chi}}{c_s}\right)\left(2\frac{c_{s,\chi}}{c_s} + \frac{1}{2}\left(\frac{(p+\rho)_{,\chi}}{(p+\rho)} - \frac{c_{s,\chi}}{c_s}\right)\right)\right]\\
&-c_s\frac{(p+\rho)_{,\chi}}{(p+\rho)}\left[\frac{(p+\rho)_{,\chi}}{(p+\rho)} - \frac{c_{s,\chi}}{c_s}\right]\Bigg\}\Bigg|_{\chi_0},
\end{aligned}
$$

$$
\mathcal{C}_{\{k2,l1,c^2\}} = -4\left\{c_s^2\left[\frac{(p+\rho)_{,\chi}}{(p+\rho)} - \frac{c_{s,\chi}}{c_s}\right]\right\}\Bigg|_{\chi_0}, \qquad \mathcal{C}_{\{k2,l1,s^2\}} = 8\left[c_s^2\frac{(p+\rho)_{,\chi}}{(p+\rho)}\right]\Bigg|_{\chi_0},
$$

$$\mathcal{C}_{\{k2,l0,sc\}} = -4\left\{c_s\left[\frac{(p+\rho)_{,\chi}}{(p+\rho)} - \frac{c_{s,\chi}}{c_s}\right]\right\}\Big|_{\chi 0}\,, \qquad \mathcal{C}_{\{k2,l2,sc\}} = 8\left[c_s^3\frac{(p+\rho)_{,\chi}}{(p+\rho)}\right]\Big|_{\chi 0}\,,$$
$$\mathcal{C}_{\{k1,l0,s^2\}} = 4c_s^2|_{\chi 0}\,, \qquad \mathcal{C}_{\{k1,l2,c^2\}} = 4c_s^4|_{\chi 0}\,, \qquad \mathcal{C}_{\{k1,l1,sc\}} = 8c_s^3|_{\chi 0}\,. \tag{43}$$

In this case we have defined $(\cdot)_{,\chi} \equiv d(\cdot)/d\chi$ and we recall that the dimensionless variables χ and κ are defined in Eq. (34). We have indicated the coefficients $\mathcal{C}_{\{k[j],l[i],[sc]\}}$ in such a way to signal that they multiply an integral in κ of κ^{n-1}/κ^j times $\sin(D_0\kappa)\cos(D_0\kappa)$ and the overall multipole coefficent is $(l+1/2)^i$. We can infer from (42) that for $n<1$ all integrals are convergent. Notice that a natural cut-off in the various integrals is introduced for those modes that enter the horizon during the radiation dominated epoch, due to the Meszaros effect that the matter fluctuations will suffer until the full matter domination epoch. Such a cut-off will show up in the gravitational potential and in the various integrals of Eq. (42) as a $(k_{eq}/k)^4$ factor, where k_{eq} is the wavenumber of the Hubble radius at the equality epoch.

A simple inspection of Eq. (42) shows one of our main results. The terms of Eq. (42) where the coefficients $\mathcal{C}$ turn out to be proportional to the sound speed c_s cause the growth of $l(l+1)I_{\Theta_l}(k\eta_{1/2}>1)/(2l+1)$, (and hence of the power spectrum $l(l+1)C_l$ through Eq. (25)), as l increases. This means that, if the sound speed of the unified dark matter fluid starts to differ significantly from zero at late times, the consequence is to produce a very strong ISW effect, and clearly this does not happen in a ΛCDM universe since $c_s^2=0$ always. This effect is easily explained by considering that the energy density of the universe in the unified models is dominated at late time by a just single fluid. Therefore an eventual appearance of a Jeans lenght (i.e. a departure of the sound speed from zero) makes the oscillating behaviour of the dark fluid pertubations under the Jeans lenght immediately visible through a strong time dependence of the gravitational potential. In fact one can verify that the scalar field fluctuations (22) are oscillating and decaying in time as $\delta\varphi \sim (k/a)\left[c_s/(\partial p/\partial X)\right]^{1/2}\sin(k\int_{\eta_{1/2}}^{\eta} c_s d\eta)$. Similar results have been discussed in the case of the GCG model in Refs. [34, 35].

We point out that the potentially most dangerous term in Eq. (42) is the one identified by the coefficient $\mathcal{C}_{\{k1,l2,c^2\}}$

$$4c_s^4|_{\chi 0}\,(l+1/2)^2\left[\int_{\frac{l+1/2}{\chi_0-1}}^{\infty}\frac{d\kappa}{\kappa}\kappa^{n-1}\cos^2(D_0\kappa)\right]\,. \tag{44}$$

Such a term makes the power spectrum $l(l+1)C_l$ to scale as l^3 until $l\approx 25$. This angular scale is obtained by considering the peak of the Bessel functions in correspondence of the cut-off scale k_{eq}, $l\approx k_{eq}(\eta_0-\eta_{1/2})$. In fact, for smaller scales, the integral identified by the coefficient $\mathcal{C}_{\{k1,l2,c^2\}}$ will decrease as $1/\ell$.

3.2.3. Intermediate case.

Now we shall briefly discuss the intermediate case that corresponds to perturbation modes that initially are outside the Jeans length and then, due to a time variation of the sound speed, they fall inside them. This corresponds to consider the range $[(l+1/2)/(\chi_0-1)]^2 < \kappa_J^2 = |\theta_{,\chi\chi}/\theta|/c_s^2 < \kappa_{eq}^2$. In this case $\kappa > (l+1/2)/(\chi_0-1)$ and so we can use the same procedure described before. Indeed when $k \sim k_J$ i.e. $c_s^2 k_J^2 \sim |\theta''/\theta|$ it can be written as follows

$$\left\{\left[c_s^2\,|\theta_{,\chi\chi}/\theta|^{-1}\right]\Big|_{\chi 0} - \left[c_s^2\,|\theta_{,\chi\chi}/\theta|^{-1}\right]_{,\chi}\Big|_{\chi 0}\left(\frac{l+1/2}{\kappa_J}\right)\right\}\kappa_J^2 = 1 \tag{45}$$

i.e.

$$\kappa_J = \kappa_J(l) = B_l + \sqrt{B_l^2 + A} \tag{46}$$

where $B_l \equiv \{[\ln(c_s^2)]_{,\chi} - [\ln|\theta_{,\chi\chi}/\theta|]_{,\chi}\}|_{\chi 0}(2l+1)/4$ and $A \equiv [|\theta_{,\chi\chi}/\theta|/(2c_s^2)]|_{\chi 0}$. We immediately see that $I_{\Theta_l}(k\eta_{1/2} > 1)$ can be divided into two parts. The first part is identical to (39) except for the upper limit of the integral. Indeed now the upper limit is $\kappa_J(l)$. In order to derive the second part we note that now the lower limit of the integral in κ is $\kappa_J(l)$ and that $u_k(\eta_k) = [C_k(\eta_J)/c_s^{1/2}(\eta_k)]\cos\left(k\int_{\eta_J}^{\eta_k} c_s(\tilde\eta)d\tilde\eta\right)$ where η_J is the conformal time when $c_s^2|_{\eta_J}k^2 \sim |\theta''/\theta|_{\eta_J}$ (η_J is function of k) and

$$C_k(\eta_J) = 2\Phi_k(0)\frac{\left[1 - \frac{\mathcal{H}(\eta_J)}{a^2(\eta_J)}\left(I_R + \int_{\eta_R}^{\eta_J} a^2(\tilde\eta)d\tilde\eta\right)\right]}{[(p+\rho)/c_s]^{1/2}(\eta_J)}\ . \tag{47}$$

4. Discussion of some examples and conclusions

In most of the UDM models there are several properties in common. It is easy to see that in Eq.(31) I_R is negligible because of the low value of a_{eq}.

Moreover in the various models usually we have that strong differences with respect to the ISW effect in the ΛCDM case can be produced from those scales that are inside the Jeans length as the photons pass through them. For these scales (which depend on the particular model) the perturbations of the unified dark matter fluid play the main role. On larger scales instead we find that they play no role and ISW signatures different from the ΛCDM case can come only from the different background expansion history. We have found that when $k^2 \gg k_J^2 = c_s^{-2}|\theta''/\theta|$ (see (19)) one must take care of the term in Eq. (42) proportional to $\mathcal{C}_{\{k1,l2,c^2\}}$. Indeed this term grows faster than the other integrals contained in (42) when l increases up to $l \approx 25$. It is responsible for a strong ISW effect and hence it will cause in the CMB power spectrum $l(l+1)C_l/(2\pi)$ a decrease in the peak to plateau ratio (once the CMB power spectrum is normalized). In order to avoid this effect, a sufficient (but not necessary) condition is that all the models have to satisfy $c_s^2 k^2 < |\theta''/\theta|$ for the scales of interest. The maximum constraint is found in correspondence of the scale at which the contribution Eq. (42) proportional to $\mathcal{C}_{\{k1,l2,c^2\}}$ takes it maximum value, that is $k \approx k_{eq}$. For example in the Generalized Chaplygin Gas model (GCG), i.e when $p = -\Lambda^{1/(1+\alpha)}/\rho^\alpha$ and $c_s^2 = -\alpha w$, we deduce that $|\alpha| < 10^{-4}$ (see Refs. [20] [34] [35] and [43]). This is also in accordance with [36] which performs an analysis on the mass power spectrum and gravitational lensing constraints thus finding a more stringnet constraint.

As far as the generalized Scherrer solution models [22] are concerned, in these models the pressure of the unified dark matter fluid is given by $p = g_n(X - X_0)^n - \Lambda$, where g_n is a suitable constant and $n > 1$. The case $n = 2$ corresponds to unified model proposed

by Scherrer [21]. In this case we find that imposing the constraint $c_s^2 k^2 < |\theta''/\theta|$ for the scales of interest we get that $\epsilon = (X - X_0)/X_0 < (n-1)\ 10^{-4}$.
If we want to study in greater detail what happens in the GCG model when $c_s^2 k^2 \gg |\theta''/\theta|$ we discover the following things:

- for $10^{-4} < \alpha \leq 5 \times 10^{-3}$, where we are in the "Intermediate case". Now $c_s^2 = -\alpha w$ is very small and the background of the cosmic expansion history of the universe is very similar to the ΛCDM model. In this situation the pathologies, described before, are completely negligible.
- When treating $6 \times 10^{-3} < \alpha \leq 1$ a very strong ISW effect will be produced and we have estimated the same orders of magnitude for the decrease of the peak to plateau ratio in the anisotropy spectrum $l(l+1)C_l/(2\pi)$ (once it is normalized) that can be inferred from the authors of [34] obtained in the numerical simulations (having assumed that the production of the peaks during the acosutic oscillations at recombination is similar to what happens in a ΛCDM model, since at recombination the effects of the sound speed will be negligible).

An important observation arises when considering those UDM models that reproduce the same cosmic expansion history of the universe as the ΛCDM model. Among these models one can impose the condition $w = -c_s^2$ which, for example, is predicted by UDM models with a kinetic term of the Born-Infeld type [22] [25] [26]. In this case, computing the integral in Eq. (42) proportional to $\mathcal{C}_{\{k1,l2,c^2\}}$ which give the main contribution to the ISW effect we have estimated that the corresponding decrease of peak to plateau ratio is about one third with respect to what we have in the GCG when the value of α is equal to 1. The special case $\alpha = 1$ is called "Chaplygin Gas" (see for example [19]) and it is characterized by a background equation of state w which evolves in a different way to the standard ΛCDM case.
From these considerations we deduce that this specific effect stems only in part from the background of the cosmic expansion history of the universe and that the most relevant contribution to the ISW effect is due to the value of the speed of sound c_s^2.

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